Engineering Notes

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Transmitted Data Volume, Orientation Accuracy, and Size of Parabolic Antenna for Spacecraft

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The use of a narrow-beam parabolic antenna for spacecraft applications provides an effective means of increasing the field strength at the ground receiving terminal. In recent years, valuable experience has been gained in the design and use of such antennas. The effectiveness of the communication link is a function of the antenna's directive gain D. If the data transmission rate does not vary automatically with the current value of D, the effectiveness of the link will be defined by the minimum assured D during one period of spacecraft oscillation relative to its mass center. This minimum D is determined by 1) the geometry of the antenna and 2) the maximum pointing error, referenced to the ground receiving point. The maximum pointing error, in turn, determines how much propellant (for an onboard jet-reaction system) and/or electric energy (for a flywheel system) will be required for the spacecraft's stabilizing system. This Note presents an analysis giving consideration to all of these factors in order to arrive at values of parabolic antenna size and pointing error that will provide the maximum quantity of transmitted data.

Optimum Antenna Size

It is known from parabolic antenna theory that

$$D = D_{\max} F(\theta x) \tag{1}$$

where θ is the angle of deviation from an antenna electric axis, and $D_{\max}$ is a maximum value of the directive gain corresponding to $\theta = 0$; $x = d/\lambda$, where λ is wavelength; and $F(\theta x)$ is a continuous differentiable function of θx, satisfying the conditions $F(0) = 1$, $dF/d(\theta x) \leq 0$ within the range of θx considered. Furthermore,

$$D_{\max} = \pi^2 K x^2 \tag{2}$$

where K is a coefficient depending on both the form and the construction of the antenna and on the parameter $y = d/f$, where f is the paraboloid focal length. Thus,

$$D = \pi^2 K(y) x^2 F(\theta x) \tag{3}$$

The theoretical dependence of K on y has been studied comprehensively; the actual values of K are of the order of 0.5 to 0.6, and the optimum value of y is in the range 1.6–3.0.

For a fixed value of θ, D reaches its maximum value under the condition

$$2F(\theta x) = (\theta x)|f(\theta x)| \tag{4}$$

where $f(\theta x) = dF(\theta x)/d(\theta x)$ and $f(\theta x) < 0$ when $x > 0$, $\theta > 0$. Solving Eq. (4) relative to (θx) we obtain $(\theta x)_{\text{opt}} = Q$, where $Q \neq Q(\theta x)$, and hence, the optimum value of x is

$$x_{\text{opt}} = Q/\theta \tag{5}$$

Consequently,

$$D_{\text{opt}} = \pi^2 K Q^2 F(Q)/\theta^2 \tag{6}$$

The function $F(\theta x)$ is determined by a complex expression in terms of Bessel functions that precludes obtaining a simple relationship for the design calculations. Therefore, the following approximation is taken:

$$F(\theta x) = D/D_{\max} = \exp[-\alpha(\theta/\theta_{0.5})^2] \tag{7}$$

where $\alpha = 0.69$ and $\theta_{0.5}$ is the value of θ at which $D = \frac{1}{2}D_{\max}$ and is obtained from the relation

$$\theta_{0.5} = \beta/x \tag{8}$$

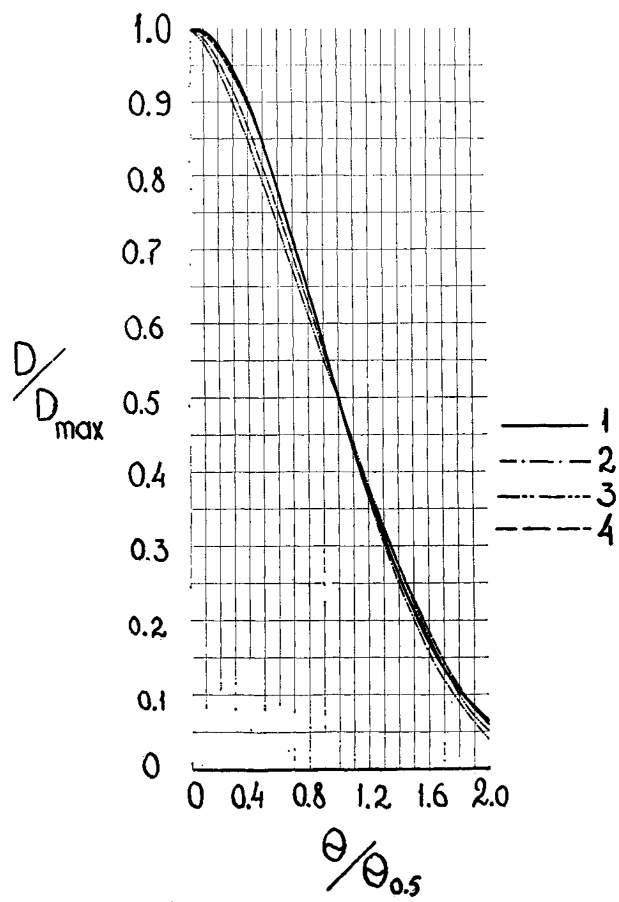

Fig. 1 Directional patterns in relative units: curve 1, approximation $D/D_{\max} = \exp[-\alpha(\theta/\theta_{0.5})^2]$, $\alpha = 0.69$; curves 2 and 3, theoretical directional patterns for two specific mutually perpendicular planes E and H, respectively, with $d/f = 2.4$; curve 4, directional pattern of the parabolic antenna used on spacecraft Zond-3.

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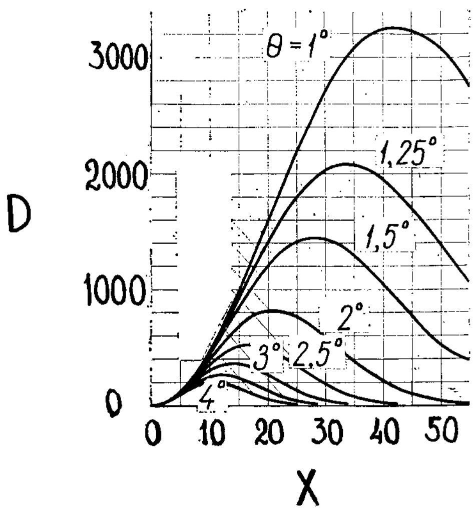

Fig. 2 $D(x)$ for various values of θ with $\alpha = 0.69$, $\pi^2K = 5$, $\beta = 0.61$.

where the coefficient β varies slightly with the form and the construction of the irradiator and the parameter y. Accurate calculations have shown that $0.5 \leq \beta \leq 0.7$ for $1.6 \leq y \leq 3.2$. Hence, it is reasonable to neglect the dependence of β on y and to assume that $\beta = 0.6$, as is usually the case in spacecraft antenna design. This approximation is shown in curve 1 of Fig. 1. The theoretical directional patterns, in relative units for the antenna with an irradiator made as an elementary vibrator with a flat reflector, are shown in curves 2 and 3 for two specific mutually perpendicular planes, E and H, respectively, with $d/f = 2.4$. The directional pattern of the parabolic antenna used on spacecraft Zond-3 is shown in curve 4. As the plots show, the approximation is quite reasonable for $\theta < 2\theta_{0.5}$.

From Eqs. (2), (7), and (8), we obtain for the approximation,

$$D = \pi^2K(d/\lambda)^2e^{-\alpha(\theta d/\beta\lambda)^2} \tag{9}$$

Then Eq. (5) and (6) can be rewritten;

$$\chi_{opt} = (d/\lambda)_{opt} = \beta/\theta\alpha^{1/2} \tag{10}$$

$$D_{opt} = \pi^2K\beta^2/e\alpha\theta^2 \tag{11}$$

In designing the antenna, d/λ (hence d) can be found for the given θ from Eq. (10) at $\alpha = 0.69$ and $\beta = 0.6$. Then, using known antenna technology techniques, an f is defined that provides the maximum K, and β in Eq. (8) is refined. If required, a second approximation can be made by correction of d_{opt} for a further refined value of β, in accordance with Eq. (10). Figure 2 shows $D(x)$ for various values of θ with $\alpha = 0.69$, $\pi^2K = 5$, and $\beta = 0.61$. An incorrect choice of d can lead to losses in the directive gain and, if $d > d_{opt}$, to an increase in antenna weight.

From the viewpoint of obtaining maximum directive gain, it is essential that the value θ be as small as possible and that d be as close as possible to the optimum size corresponding to this θ. For those cases where the specific purpose of the spacecraft does not allow this freedom of choice, it would be expedient to take the given D and select a d that would achieve this gain with the largest permissible θ. For this case from Eq. (9) we get

$$\theta = [-\alpha^{-1}\ln(D/\pi^2Kx^2)]^{1/2}\beta/x \tag{12}$$

This expression has a maximum value when

$$x = x_{opt} = (De/K)^{1/2}/\pi \tag{13}$$

Then, if it is assumed that the optimal parabola is found for the given value θ with $x = x_{opt} = \beta/\theta\alpha^{1/2}$ for this parabola,

$$\theta_{0.5} = \beta/x_{opt} = \alpha^{1/2}\theta \cong 0.83\theta \tag{14}$$

Comparison of the D_{opt} found from Eq. (11) with the D_{max} corresponding to the position of the reception point on an optimal parabolic antenna axis results in

$$D_{opt} = D_{max}/e \tag{15}$$

Equations (14) and (15) can be considered as defining the optimal parabola at the given angle θ for an assumed approximation of directional pattern.

Effects of Stabilization Energy Requirements

The quantity of propellant or electric energy required for orientation depends on required accuracy. Therefore, improvement in orientation accuracy can lead to the necessity of reducing data transmission time if onboard energy stores are limited. If the data transmission rate is invariable, the amount of data transmitted in time τ can be expressed by

$$M = AD\tau \text{ (in arbitrary units)} \tag{16}$$

where A is a parameter depending on the characteristics of the on-board and ground-based radio systems as well as the distance to the vehicle and is constant for the time τ, and D is an antenna directive gain assured for the time τ. In the paragraphs that follow two cases will be considered: the case of the optimal parabolic antenna and the case where specific antenna gain is dictated.

For the optimal parabolic antenna, d and θ are related by Eq. (5), and Eq. (6) gives

$$M = \pi^2AKQ^2\tau F(Q)/\theta^2 \tag{17}$$

For this discussion two modes of vehicle orientation will be considered, assuming that the parabolic antenna is fixed relative to the vehicle for the time τ. The first mode pertains to the use of an onboard gas-jet system that sends controlling impulses to the vehicle with angular deviations corresponding to the boundaries of the orientation system deadband. Here the weight of the propellant required for the time τ is

$$G_{pm} = B_1\tau/\varphi \tag{18}$$

where B_1 is a parameter depending on both vehicle moments of inertia and characteristics of the orientation and gas-jet systems, and φ is one-half of the deadband angular width, or a maximum deviation of the direction to the ground station from the deadband axis in a steady auto-oscillation mode. Equation (18) is satisfied for identical maximum deviations over the corresponding pilot channels.

The second mode includes, besides the onboard gas-jet system, a flywheel system of stabilization with the kinetic moments parallel to the vehicle axis. At angular deviations that are determined by the deadband boundaries, a permanent moment of appropriate sign is applied to the corresponding flywheel. This moment changes the flywheel's angular velocity ω, and, consequently, both the angular velocity and angular position of the vehicle. This change in ω ceases when the vehicle is oriented to the interior of the deadband. Typically, for this mode electric power is expended only on the change of ω in a steady auto-oscillation mode, and propellant is expended only on a periodic elimination of the results of external disturbing moments, the effect of which can lead to an unlimited increase in ω. The consumption of propellant does not then depend on orientation accuracy. The electric power required for the vehicle orientation during τ can be approximated as

$$P = B_2\tau/\varphi \tag{19}$$

where B_2 depends on the moments of inertia of the vehicle and the flywheel, as well as on characteristics of the electrical power systems, and φ is of the same value as expressed in Eq.

(18). Values φ and θ in Eqs. (17–19) are related by

$$\theta = \varphi + \delta \tag{20}$$

where δ is the maximum error between the axis of the deadband and the parabolic antenna electric axis; δ is a function of spacecraft design, method of achieving correct vehicle orientation, accuracy of determining antenna electric axis, and electric and dynamic parameters of the pointing system. For present vehicles, δ is 10–60 arcmin.

It should be noted that the most unfavorable case is considered when the plane passing through the antenna electric axis and the deadband axis (plane of angle δ) coincides with the plane of maximum deviation in the orientation system.

Next, let us consider a "specific information" parameter $\bar{M}$, defined as the ratio of the amount of data transmitted during τ to either the weight of propellant expended or the amount of electric energy consumed during τ (for the first and second modes, respectively). In the optimal parabolic antenna case we find from Eqs. (17–20) that

$$\bar{M} = \pi^2[AKQ^2F(Q)/B_i]\cdot[\varphi/(\varphi + \delta)^2] \tag{21}$$

where $i = 1$ or 2, according to the given mode, and $\bar{M}$ is maximum when

$$\varphi = \varphi_{\text{opt}} = \delta \tag{22}$$

Two points should be noted; 1) since the general Eq. (6) is used to derive the expression in Eq. (21), the obtained result does not depend on the type of directional pattern approximation but is specific only to the considered optimal parabolic antenna, and 2) this conclusion does not depend on the scheme for orientation to the Earth; i.e., whether it is accomplished by means of orientation to the sun and reference star or directly to the Earth by optical or radio-bearing techniques.

Figures 3 shows m vs φ for different values of δ, where $m = \varphi/(\varphi + \delta)^2$. It is seen that $\bar{M}$ essentially depends on φ, at small values of δ, and an incorrect choice of orientation accuracy can lead to a significant decrease in $\bar{M}$ when $\delta = 0.25°$ to $0.5°$. On the other hand, $\bar{M}$ is insensitive to φ when $\delta = 1°$ to $2°$.

For the given directional pattern approximation, with an optimal orientation accuracy as determined from Eqs. (10), (20), and (22),

$$d_{\text{opt}} = \lambda\beta/2\delta\alpha^{1/2} \tag{23}$$

For some examples it will be found that φ_{opt}, as defined by Eq. (22), cannot be realized by a specified time moment. Therefore, the concept of technically realizable orientation accuracy $\bar{\varphi}$ must be introduced. If φ_{opt} appears to be less than $\bar{\varphi}$, it would be expedient to admit the value $\bar{\varphi}$. However, to maximize M in this case, d should be chosen optimal for $\bar{\varphi} + \delta$.

Finally, another possible variant is that φ_{opt} is realized, but the d determined by Eq. (23) cannot be permitted because of design or weight considerations. For such cases, a more general equation for φ_{opt} at a limited diameter $\bar{d}$ can be derived, based on the approximation of Eq. (7). From Eqs. (9), (16), and (18–20) we obtain

$$\bar{M} = A\pi^2K\varphi(\bar{d}/\lambda)^2 \exp[-\alpha(\bar{d}/\lambda)^2(\varphi + \delta)^2/\beta^e]B_i \tag{24}$$

$\bar{M}$ reaches a maximum value when

$$\varphi = \varphi_{\text{opt}} = \tfrac{1}{2}\{[\delta^2 + 2\beta^2(\lambda/\bar{d})^2/\alpha]^{1/2} - \delta\} \tag{25}$$

Equation (22) for an optimal parabolic antenna can be easily obtained from Eq. (25) if $\bar{d}/\lambda$ is taken as $\beta/\alpha^{1/2}(\varphi_{\text{opt}} + \delta)$, according to Eq. (10), and the resulting expression is solved relative to φ_{opt}; i.e., the use of Eq. (22) for the approximation of Eq. (7) is a specific case of Eq. (25).

Note that φ_{opt} defined from Eq. (25) always exceeds $\bar{\varphi}$. Actually, if $\bar{d}$ is assumed to be less than optimal, then according to Eq. (23) $\bar{d}/\lambda < \beta/2\delta\alpha^{1/2}$. Under this condition, Eq. (25) gives $\varphi_{\text{opt}} > \delta$. However, δ equals the optimal orientation accuracy that is realized according to the assumption.

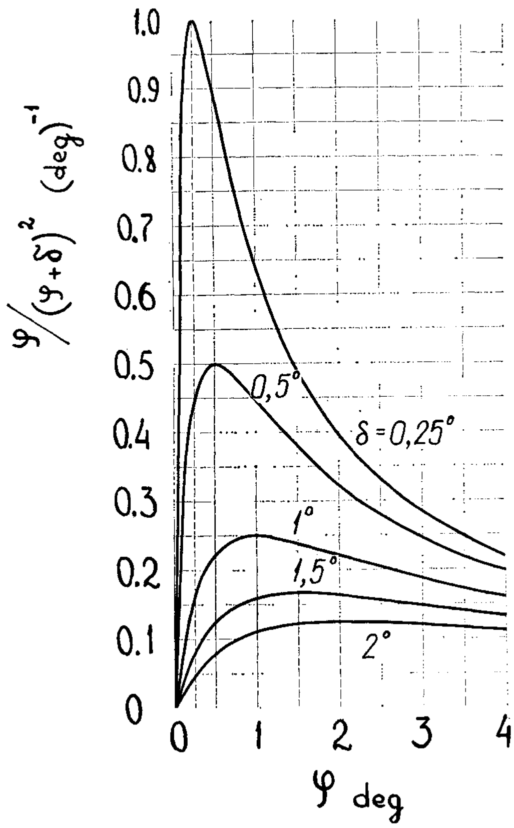

Fig. 3 $\varphi/(\varphi + \delta)^2$ **vs** φ **for different values of** δ.

Applications of Results

In conclusion, let us consider the practical applications of the obtained results for three situations:

1) Gas-jet stabilization only. In this case,

$$M = \bar{M}G_{pm} \tag{26}$$

so that the maximum value of $\bar{M}$ corresponds either to the maximum volume of data transmitted for the given propulsion mass G_{pm} or to the minimum required G_{pm} for a given volume of data. Obviously, maximum specific information will be realized only with the most effective use of propellant. Eliminating φ from Eqs. (17) and (18) and taking into account Eq. (20), the following relationships can be obtained:

$$M = C\tau/[(B_i\tau/G_{pm}) + \delta]^2 \tag{27}$$

or

$$G_{pm} = B_i\tau/[(C\tau/M) - \delta]^{1/2} \tag{27a}$$

where $C \neq C(\tau)$. It follows that the optimal value of φ realizing the maximum of $\bar{M}$ [Eq (21)] corresponds to the optimal value of τ that realizes the maximum of M for the given G_{pm} per Eq. (27) or the minimum of G_{pm} for the given M per Eq. (27a). From Eq. (27) it also follows that if τ is specified, there is no optimal φ, because as φ decreases (G_{pm} increases), M rises. In the latter case, a minimum allowable φ should be taken which can be determined either by the technical capabilities of the orientation system or by G_{pm}. So it is expedient to install an optimal parabola for the indicated value of φ, as it provides maximum directive gain.

2) Gas jets (to correct external disturbance torques) plus flywheel stabilization on system powered by an electric battery. In this case,

$$M = \bar{M}P \tag{28}$$

where P is proportional to the battery capacity. This variant resembles case 1, except that the battery capacity, rather than the propellant reserve, is taken into account; i.e., the optimal φ providing the maximum of $\bar{M}$ also has a technical significance here.

3) Gas jets plus, flywheel, stabilization system powered by solar battery. In this case,

$$\bar{M} = (dM/dt)/N \quad (29)$$

where dM/dt is the data transmission rate;

$$dM/dt = C/(B_3/N + \delta)^2$$

N is the average power obtained from the solar cells to maintain orientation;

$$N = B_3/\varphi$$

and B_3 is a parameter depending on the vehicle and flywheel moments of inertia as well as on characteristics of the orientation and power supply systems. In this case a quest for maximum $\bar{M}$ by choice of the optimal φ has no technical significance, because dM/dt increases as long as φ decreases (N increases). Thus, as in the first case at τ = constant, it is expedient to take the minimum allowable value φ determined by the technical capabilities of the orientation system or by the limiting capabilities of the solar battery (weight or area). The optimal parabolic antenna for this value of φ will provide maximum D and maximum M under the given limitations.

Flight Measurement of Aerodynamic Coefficients on a Bomb

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Nomenclature

c_1 to c_8 = least-squares coefficients
C_m, C_n = static normal and side moment coefficients, respectively
C_Z, C_Y = static normal and side force coefficients, respectively
M = Mach number
$OXYZ$ = bomb body axes
p,q,r = angular velocity components
t = time from release
α,β = angle of pitch and yaw, respectively
θ = total angle of incidence
ϕ = angular orientation of body relative to plane of incidence; $\tan\phi = \tan\beta/\tan\alpha$

Introduction

A JOINT research program was initiated in 1960 by the Royal Aircraft Establishment (RAE) in the United Kingdom and the Weapons Research Establishment (WRE) in Australia to study the ballistic performance of bombs stabilized by fixed cruciform fins.[1] The program was extended in 1964 in the United States on a tripartite basis in collaboration with the Naval Ordnance Laboratory (NOL) in order to deal with bombs stabilized by split skirts and freely spinning tails.[2-4] To complete the program, two bombs stabilized by fixed cruciform fins were dropped during 1968 to study two types of flight instability, namely catastrophic yaw and Magnus instability.

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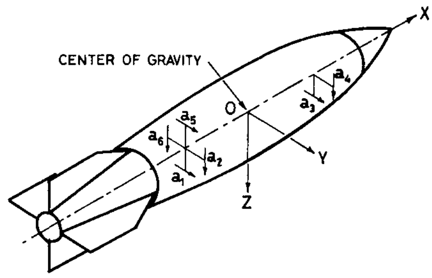

Fig. 1 Positions on bomb of six-linear accelerometers.

An aim of the program has been to determine the aerodynamic stability coefficients as a function of M, θ, and ϕ. A convenient mathematical model of the forces and moments, which makes use of the properties of rotational and reflectional symmetry of a vehicle with fins, has been developed by Maple and Synge.[5] The present Note describes the derivation of aerodynamic force and moment coefficients on the M823 research store using the Maple and Synge theory, and is based on three Technical Notes[6-8] which supply more detail as well as the computer program used. Because of insufficient instrumentation and inaccurate data, the axial forces and moments could not be analysed, so that only lateral force and moment coefficients are derived. The method has been applied to bombs stabilized by split skirts and freely spinning monoplane and cruciform tails, but is here confined to bombs stabilized by fixed cruciform fins, and results are given only for the catastrophic yaw experiment.

Instrumentation

The Euler equations of motion show the various quantities required in order to determine the aerodynamic forces and moments.[9] Ground cameras together with meteorological data provided the necessary trajectory data. Given the mass and moments of inertia of the bomb, the problem was to select and position instruments on the bomb which would most easily and accurately record the required quantities using a telemetry system.[6]

The instruments fitted were a differential pressure incidence meter to obtain α and β, four laterally positioned linear accelerometers, three rate gyroscopes with mutually perpendicular axes (one along OX in Fig. 1), and a strain gauge mounted in the tail section to record roll torque. This selection would enable a comparison of forces and moments derived using various combinations of instrument measurements. Good agreement would serve to verify the reliability of these measurements. A longitudinally positioned accelerometer was not fitted because previous experiments proved unsuccessful.

By differentiating incidence meter measurements, pitch and yaw rates q and r were derived as an alternative to direct measurement by rate gyroscopes. Linear accelerometers were used both directly to give the forces and indirectly for deriving angular accelerations $\dot{q}$ and $\dot{r}$, thus providing an alternative to differentiated rate gyroscope measurements. To derive the forces and moments without using rate gyroscopes, at least five and preferably six suitably positioned linear accelerometers were required (labelled a_1 to a_6 on Fig. 1). On the vehicle being studied, a_5 and a_6 were omitted, but by using an iterative approximation procedure,[6] the lateral forces and moments could be derived. Because roll acceleration $\dot{p}$ could not be obtained to sufficient accuracy from either the strain gauge, ground cameras, or appropriate rate gyroscope, the axial moments could not be analysed.

Mathematical Model

Dependence of the derived aerodynamic forces and moments upon θ, ϕ, and p, q, and r is determined by fitting the